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Dissipative cosmology with decaying vacuum energy

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Abstract : The present model is a generalization of the model of Freese *et al* (1987) by the inclusion of a dissipative term which results in avoiding the initial singularity and incorporates particle production. It starts exponentially from a non-singular hot origin in the infinite past and expands more and more slowly and finally reduces to the Murphy model in the matter dominated phase of evolution. It is found that the inclusion of vacuum energy lengthens the period of inflation in the early stages of evolution which may be instrumental in solving horizon, flatness or monopole problems.

Keywords : FRW models, vacuum energy, particle production

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1. Introduction

In order to solve the cosmological constant problem, Freese *et al* [1] have invoked the idea of a dynamically decaying vacuum energy density and thereby presented a flat big bang model of the universe in terms of a parameter x defined by

$$x = \rho_v / (\rho_v + \rho_r) \tag{1.1}$$

i.e. the ratio of vacuum (ρ_v) to the sum of vacuum and radiation (ρ_r) energy densities. From a detailed discussion, they find that x should be a constant lying between 0 and 1 to obtain a genuinely new cosmology. However x may assume two different constant values in the different phases—one in the radiation dominated and the other in the matter dominated phase of evolution. Vacuum is primarily coupled with radiation and decays throughout the evolution to give rise to entropy or matter.

The obtained model is described as

$$\left.\begin{aligned} R &\sim t^{\frac{1}{2(1-x)}} \\ \rho_v &= \frac{3x}{32\pi G(1-x)^2 t^2} \end{aligned}\right\} \text{in the radiation dominated universe,} \tag{1.2}$$

$$\left.\begin{aligned} R &\sim t^{2/3} \\ \rho_v &\sim t^{-8(1-x)/3} \end{aligned}\right\} \text{in the matter dominated universe.} \tag{1.3}$$

and

However, this model does not incorporate particle production or dissipation. As we know, both effects play important roles in cosmic expansion [2,3] and may, in principle, have notable observational consequences [4]. Classically, bulk viscous stresses arise because the expansion of the universe is continuously trying to pull the fluid out of thermal equilibrium. Bulk viscosities also appear as a phenomenological description of quantum particle production near the Planck time and drive inflation. Inflation occurs if a weakly coupled scalar field is displaced from its equilibrium state and the subsequent evolution towards the new equilibrium state proceeds sufficiently slowly [5]. It may be noted that the occurrence of inflation demands the violation of strong energy condition [5]. It is necessary for inflation to come to an end if the universe is to resume the present Friedmann like expansion. This may be achieved by the rapidly varying coupling of the inflation fields to other matter fields, oscillating about the equilibrium state, which gives rise to particle production and hence the subsequent decay of the inflation field. This decay can also be described macroscopically by dissipation leading to particle production [5]. In this regard, Barrow [5] has pointed out that particle creation due to the non-adiabatic decaying of the field driving slow rollover inflation can macroscopically be described by the viscous cosmological model of Murphy [3] which can be obtained by considering the flat FRW model with a bulk viscosity coefficient proportional to the energy density and a perfect fluid with a barotropic equation of state ($p = [\gamma - 1]\, \rho$, $\gamma =$ constant). The evolution of the model is then described by

$$\ln R^{3\gamma/2} + CR^{3\gamma/2} = 3\gamma H_0 t/2, \; C = \text{constant} > 0, \tag{1.4}$$

which indicates that the model evolves from a deSitter state $R = e^{H_0 t}$ at $t = -\infty$ to a zero curvature Friedmann state with $R \sim t^{2/3\gamma}$ as $t \to +\infty$. This model is non-singular at the cost of energy conditions as the strong energy condition is violated as H becomes sufficiently close to H_0.

Our aim, in the present paper, is to explore the consequences of generalizing the model of Freese *et al* by the inclusion of the above mentioned effects in a manner that preserves the homogeneity and isotropy.

2. The field equations

We begin our analysis by considering an isotropic, homogeneous spacetime with flat spatial sections :

$$ds^2 = -dt^2 + R^2\left\{dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right\}. \tag{2.1}$$

The inclusion of viscosity in the model can be done by supplementing the hydrostatic pressure p with a dissipative pressure

$$p' = -\xi\theta, \tag{2.2}$$

provided the motion is one of pure expansion. Here ξ is the coefficient of bulk viscosity and $\theta\,(= 3(\dot{R}/R))$ represents the expansion of the fluid. Incorporated into Einstein's field equations, the term $-\xi\theta$ can be looked at two ways : (i) as a viscous pressure and (ii) as a creation pressure. As both the processes are scalar ones, they may take place simultaneously too [6].

With (2.2), the usual energy momentum tensor of the perfect fluid is modified to the form

$$T^{ij} = (\rho + p - \xi\theta)v^i v^j + (p - \xi\theta)g^{ij}, \tag{2.3}$$

where ρ, the energy density of the non-vacuum component, is the sum of rest mass and radiation energy densities *i.e.*

$$\rho = \rho_m + \rho_r \tag{2,4}$$

whereas the pressure p of the non-vacuum component is that of radiation;

$$p = p_r = (1/3)\rho_r. \tag{2.5}$$

With vacuum density ρ_v, playing the role of an effective cosmological constant, the Einstein field equations can be written as

$$R^{ij} - \frac{1}{2}Rg^{ij} = -8\pi G\left[T^{ij} + T^{ij}_{(v)}\right], \tag{2.6}$$

where $T^{ij}_{(v)} = -\rho_v g^{ij}$ is the energy momentum tensor of vacuum with the underlying implication that the pressure of vacuum $p_v = -\rho_v$. The divergence of eq. (2.6) indicates that the conserved quantity, in this case, is the sum of the energy momentum tensors of matter and vacuum. With (2.3), the Einstein field eq. (2.6), when incorporated in the spacetime (2.1), are obtained as

$$8\pi G(\rho + \rho_v) = 3H^2, \tag{2.7}$$

$$8\pi G(p - 3\xi H - \rho_v) = -2\dot{H} - 3H^2. \tag{2.8}$$

We assume that the coefficient of viscosity ξ is given by [2,3,7]

$$\xi = \alpha\rho. \tag{2.9}$$

with α, *a* constant.

In the following sections, we investigate the dynamical consequences and evolution of the model in the different phases of evolution starting with the initial phase of pure radiation.

3. The very early universe

In the early pure radiation era (where $\rho_m = 0$), eqs. (1.1), (2.4) and (2.7) obtain

$$\rho_v = \{x/(1-x)\}\rho_r = \{3x/(8\pi G)\}H^2. \tag{3.1}$$

Using (2.4), (2.5) and (2.9) in (2.8) and thereby eliminating ρ_r with the help of (3.1), we obtain

$$2\dot{H} + (1-x)(4-9\alpha H)H^2 = 0. \tag{3.2}$$

This equation is very similar to the one obtained by Murphy [3] for $\gamma = 4/3$, though the essential differences are there due to the appearance of vacuum energy density ρ_v, and corroborates the model of Freese *et al* for $\alpha = 0$.

Inserting the inflationary solution

$$R = R_0 e^{at}, \tag{3.3}$$

in (3.2), we observe that (3.3) is a solution of (3.2) provided: (i) $x = 1$ or (ii) $a = 4/9\alpha$. As the case (i) is ruled out at the level of the bounds of ref. [8], we find that (3.2) has an inflationary solution (3.3) (with a constant expansion rate with $H = a$ [3]) wherein the inflation is being driven by dissipation. However, the solution (3.3) is unstable as t although it is a stable attractor as $t \to -\infty$ [5].

The general solution of (3.2) is

$$H = H_0 / \left\{1 + CR^{2(1-x)}\right\}, \tag{3.4}$$

where $H = H_0 \equiv 4/9\alpha$ at $R = 0$ and C stands for a constant of integration. A further integration of (3.4) obtains

$$2(1-x)H_0(t-t_0) = \ln R^{2(1-x)} + CR^{2(1-x)}, \tag{3.5}$$

where t_0 is another constant of integration. A similar classification of solutions corresponding to the different values (zero, negative and positive) of C can be made as done in the Murphy model. For $C = 0$, eq. (3.5) reduces to the inflationary solution (3.3) with $R_0 = e^{H_0 t_0}$ but becomes unstable as a slight change of C from 0 disturbs significantly the corresponding time dependence of R. For $C < 0$, t cannot increase beyond a finite value (see the corresponding analysis in [3]). With $C > 0$, (3.5) represents a viable solution in which the expansion continually slows down but never reverses. The model universe thus started from a non-singular hot origin in the infinite past. A critical study of equation (3.5) indicates that when R is sufficiently small, one has $\left|\ln R^{2(1-x)}\right| >> CR^{2(1-x)}$ (as $x < 1$) and eq. (3.5) reduces to the inflationary solution (3.3), as is the case in [3]. On the other hand when $CR^{2(1-x)} >> \left|\ln R^{2(1-x)}\right|$, one obtains

$$R = \left[(2(1-x)H_0)/C\right]^{1/2(1-x)} t^{1/2(1-x)}, \tag{3.6}$$

so that
$$\rho_v = 3x/\left\{32\pi G(1-x)^2 t^2\right\}. \tag{3.7}$$

Eq. (3.6) has the same time dependence of R as in the model of Freese *et al* in this era and eq. (3.7) is the same as the corresponding one in this model. However, the solutions (3.3), (3.4), (3.5) and (3.6) exist only in the presence of dissipation. For $\alpha = 0$, they become singular and the general solution of eq. (3.2) in this case reduces to the big bang one obtained in the model of Freese *et al.* Thus, the inclusion of dissipation in the model of Freese *et al* reflects in a power of removing the initial singularity.

It is to be noted that the solution (3.6) exhibits an inflationary (general) character (*i.e.* $\ddot{R} > 0$) for $1 > x > \frac{1}{2}$ and a deflationary one ($\ddot{R} < 0$) for $0 < x < \frac{1}{2}$ whereas the corresponding solution in Murphy's model is certainly a deflationary one. Thus the inclusion of vacuum in the viscous model of Murphy may have an effect of lengthening the inflationary period in the early stages of evolution which might be instrumental in solving the horizon, flatness of monopole problems.

From eqs. (3.1) and (3.2), we find that the creation rate of radiation per unit volume is given by

$$(1/R^3)(d/dt)(\rho_r R^3) = \{3(1-x)/8\pi G\}\{9\alpha H(1-x) + (4x-1)\}H^3, \tag{3.8}$$

which is very high in the phase of inflation.

4. The present universe

As the most of vacuum energy must have been exhausted in generating radiation during the inflationary phase, it is reasonable to neglect ρ_v and also ρ_r compared to ρ_m, in the present pressureless phase of evolution, in eqs. (2.7) and (2.8). This gives

$$\rho_m = (3/8\pi G)H^2, \tag{4.1}$$

and

$$2H + 3H^2 - 24\pi G\alpha H\rho_m = 0, \tag{4.2}$$

which obtain

$$2\dot{H} + 3H^2(1-3\alpha H) = 0. \tag{4.3}$$

This is the same equation as obtained by Murphy for a dust universe and has solutions

$$H = 1/\left\{3\alpha(1+CR^{3/2})\right\}, \tag{4.4}$$

and

$$R = (1/2\alpha C)^{2/3}t^{2/3}, \text{ with } C > 0. \tag{4.5}$$

Although the deflationary solution (4.5) becomes singular for $\alpha = 0$, it has the same time variation of R as in the model of Freese *et al* in the matter dominated era.

An expression for the present value of the deceleration parameter $q = -(R\ddot{R}/\dot{R}^2)$, can be obtained from eqs. (4.3) and (4.4) :

$$q_p = \frac{1}{2} - (9\alpha/2)H_p. \tag{4.6}$$

Although we have no way of calculating the coefficient of viscosity α, however, we can make an estimate of the present value of deceleration parameter q_p by assuming that the viscosity is now negligible [3]. This gives $q_p \approx 0.5$ as in the standard model. The present matter dominated phase is endowed with a generation rate of particles given by

$$(1/R^3)\,(d/dt)\,(\rho R^3) = \{(27\alpha)/(8\pi G)\}H^4, \tag{4.7}$$

which is negligible at present as is the case in the standard model. Although the age of the universe is infinite in this model, however for practical purposes, the evolutionary time for the universe can be taken as that predicted by the standard model [3].

5. Concluding remarks

With a vacuum energy density that decays with expansion as the radiation energy density, we have developed a homogeneous isotropic model of the universe which incorporates viscous heating and/or particle production. The model starts from a non-singular hot origin with inflation and expands more and more slowly and finally reduces to the standard model in the matter dominated era as viscosity dies out.

The present work might be looked at two ways. On one hand, it is a generalization of the model of Freese *et al* by the inclusion of a dissipative term which results in avoiding the initial singularity and the resulting model is incorporated with particle production. On the other hand, it is a generalization of the viscous model of Murphy by the inclusion of an effective cosmological 'constant' which decays in time. This results in lengthening the evolutionary period of the model in the very early universe.

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